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Quantum Pattern Recognition with Probability of 100%

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Abstract In recent years there has been an increasing focus on the quantum pattern recognition, especially quantum multi-pattern recognition in computer science. This paper presents a new quantum multi-pattern recognition method based on the improved Grover's algorithm. This method not only details the process of quantum multi-pattern recognition using several unitary operators, but also introduces a new design scheme of initializing quantum state and quantum encoding on the pattern set. If the rate of the number of the recognized pattern on the total patterns is over $1/3$, this new method can recognize multi-pattern simultaneously with the probability of 100%. Mathematic calculations and simulation results on the case show that the proposed method can accomplish multi-pattern recognition with the probability of 100%. However, the recognition probability of other pattern recognition methods is impossible to reach 1.

Keywords Multi-pattern recognition · Improved Grover algorithm · Probability of 100%

1 Introduction

In recent year, quantum computing has been gradually paid more and more attention. In 1982, Feynman indicated that a new computer built on quantum mechanics is likely to have more efficiency in solving non-polynomial hard problem than classical computer [1]. On the base of Feynman' working, Deutsch, in 1985, pointed out that coherent superposition of quantum state may realize quantum parallel computing [2]. In 1994, it is well known that Shor [3] proposed a polynomial time algorithm for prime factorization using quantum computers. And that factoring integer is generally thought to be hard on classical computer. In 1996, Grover presented Grover's algorithm [4] for the database search, which can find a marked element in an unsorted database of size N, in $O(\sqrt{N})$ steps (compared to $O(N)$ steps using the classical computation).

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There is a little application based on Grover algorithm in the field of quantum computation. For example, in 2000, Ezhov and Ventura proposed a model of quantum associative memory with distributed queries [5, 6]. In 2001, it was also Ezhov who realized pattern recognition using quantum entanglement and Everett interpretation of quantum mechanics [7]. In 2002, Ventura firstly applied Grover's algorithm to pattern classification of only one quantum state [8]. Because of directly making use of Grover search algorithm, his pattern classification method is very simple. In 2007, Li Pan-Chi et al. proposed an improved method of the Grover algorithm. They constructed the unitary matrix that inverts the phase of the desired quantum state with 90°, and that Grover algorithm inverts the phase with 180° [9]. This paper presents a quantum multi-pattern recognition method with the probability of 100% by making use of the improved Grover search algorithm. This method introduces a new design scheme of initializing quantum state and quantum encoding.

The rest of the paper is organized as follows: Sect. 2 briefly describes the basic concept of quantum theory. Section 3 introduces the improved Grover quantum search algorithm. In Sect. 4, we introduce the method of multi-pattern recognition with the probability of 100%. Section 5 shows the comparing results with other method of pattern recognition. In Sect. 6, we summarize and expect the work.

2 Basic Quantum Theory [10]

As the smallest unit of information processing, a quantum bit or qubit is a quantum system whose states lie in a two dimensional Hilbert space. Note that a qubit can be in "1" state, "0" state or simultaneously in both (superposition). The state of a qubit can be represented as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β specify the probability of the corresponding states, and $|\alpha^2| + |\beta|^2 = 1$. Use is made here of the Dirac bracket notation, where the ket $|\cdot\rangle$ indicates a column vector and the $\langle\cdot|$ an analogous to the complex conjugate transpose of the ket. The state of a qubit can be changed by unitary transformation (or quantum operator), which is of central importance in quantum mechanics, e.g., the closed quantum mechanical systems transform only via unitary transformations that preserves quantum probabilities.

2.1 Linear Superposition

The state $|\varphi\rangle$ of a general quantum system can be described by the linear superposition of the basis states $|\phi_i\rangle$ as$|\varphi\rangle = \sum_i c_i|\phi_i\rangle$, where c_i is a complex and $\sum_i |c_i|^2 = 1$. Using quantum operators, an eigenvalue equation can be written as $A|\phi_i\rangle = a_i|\phi_i\rangle$, where A is an operator and a_i is the eigenvalue. The solutions $|\phi_i\rangle$ to such equations are called eigenstates and can be used to construct the above basis of the Hilbert space. In quantum computation, the target problem is first "translated" to the language of quantum states and then quantum operators are applied to drive the system to a final state where the solution can be identified with a high probability.

2.2 Quantum Measurement

A result of quantum measurement (QM) is that if a system that is in a linear superposition of states interacts in any way with its environment, the superposition is destroyed. This loss of coherence is called decoherence and is governed by the wave function $|\varphi\rangle$. The coefficient c_i is called probability amplitudes, and $|c_i|^2$ gives the probability of $|\varphi\rangle$ collapsing into the ground state $|\phi_i\rangle$ if it decoheres. Note that the wave function $|\varphi\rangle$ describes a real physical system that must collapse to exactly one ground state.

3 Improved Grover Quantum Search Algorithm

Before we describe this part, we assume that the reader is familiar with the basics of quantum circuits, especially the standard Grover quantum search algorithm that inverts the phase of 180° of the quantum state. The essence of quantum search approach is to construct an iteration scheme consisting of unitary transformations which change the wave function in such a manner that the amplitude of the desired state increases and the amplitudes of other states decrease. Then the collapse of wave function (choice of entry in the database) will give, with a certain probability, the desired marked entry. But the more the number of the identified pattern, the smaller the successful probability, so we need to improve the Grover algorithm. The improved algorithm can be described as follows:

Step 1 Initializing state by applying $H^{(k)} = H \otimes H \otimes \cdots \otimes H$ (k is the number of Hadamard gates) to the state $|0\cdots 0\rangle$ and gets $|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle (N = 2^k)$as an equiprobable superposition of all entries, where k and $|x\rangle$ also denote the number of qubits and the ground state respectively.

Step 2 Constructing a projection $U_a = I - (1-i) \sum_{l=1}^{M} |a_l\rangle\langle a_l|$ by using desired state $|a_l\rangle$. The operator U_a inverts the phase of the state $|a_l\rangle$ with 90°. Where i is an imaginary number and M is the number of desired patterns and it is a large number, also meets$M > \frac{1}{3}N$, or else this algorithm has little significance.

Step 3 Unitary matrix $U_s = (1+i)|s\rangle\langle s| - iI$ inverts the amplitudes of all states around their average value.

Step 4 It has been shown that after $T \cong \frac{\pi}{4}\sqrt{N/M}$ iterations (Step 2 and Step 3) the amplitude of $|a_l\rangle$ becomes above 50% while the amplitudes of other states almost vanish.

4 Multi-Pattern Recognition with Probability of 100%

Due to the power of quantum parallelism, this paper presents a method that can recognize quantum multi-pattern with the probability of 100% based on the improved Grover algorithm.

4.1 Learning Pattern Set

Definition 1 Pattern set $P = \{(x_1^n x_2^n \cdots x_i^n)\}, x \in \{0, 1\}$, where i is the number of binary bit of a pattern and n is the total number of patterns; Spurious set $S = \{(x_1^s x_2^s \cdots x_i^s)\}$, where s is the total number of spurious states in the S, and $s = n$, i.e. the number of the patterns equals the number of the spurious state. But $s + n < 2^i$. Therefore, the way of initializing state is modified by following formula (1) that is different from Grover's method.

$$|\varphi\rangle = \frac{1}{\sqrt{2n}} \sum_{x_i \in P,S}^{2n} |x_1 \cdots x_i\rangle . \tag{1}$$

From formula (1), the patterns in the P and S are regarded as the basis states of linear superposition states. And their coefficients do not equal to zero, whereas the coefficients corresponding to the patterns that are not in the P and S are zero in the process of quantum encoding. For example, in the P, taking $i = 3, n = 3$, we may select $P = \{000, 010, 100\}$

and $s = 3$ in the S. Therefore, formula (2) is the original quantum state of this example on the base of the formula (1) (the spurious states in the S are selected arbitrarily).

$$|\varphi\rangle = \frac{1}{\sqrt{2\times 3}}(|000\rangle + |010\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle). \tag{2}$$

Formula (2) can also be denoted with their corresponding indices because Grover algorithm can only search the indices of the database item and cannot search the contents themselves (seeing formula (3)).

$$|\varphi\rangle = \frac{1}{\sqrt{6}}(1, 0, 1, 0, 1, 1, 1, 1). \tag{3}$$

Where 1 not only denotes being of its corresponding basis state but also represents the index of corresponding state and the basis state corresponding to 0 is not being. For example, the first 1 represents the index of the state $|000\rangle$. And that the distribution of spurious states is arbitrary.

From above formulas, we can see that $|\varphi\rangle$ contains all patterns in the P and S. So we have completed the learning of the pattern set.

4.2 The Method of Pattern Recognition

Using the proposed pattern set learning method, we can conduct pattern recognition. After finite improved Grover iterations, we can get the desired result with the probability of 100% when the quantum system is measured. The proposed method of pattern recognition can be given as follows:

Step 1 Following the method recommended in 4.1, we can form $|\varphi_0\rangle$ by learning pattern set. But the way of quantum coding contains two cases:

(1) if $n < \frac{1}{2}(2^i)$, we require i qubits to encode $|\varphi_0\rangle$ into the linear superposition state;
(2) if $\frac{1}{2}(2^i) < n < \frac{1}{2}(2^{i+1})$, $(i+1)$ qubits are needed for quantum encoding. Obviously, quantum encoding of formula (2) satisfies the first case, therefore, we use three qubits to encode formula (2).

Step 2 Constructing a unitary operator that is applied to the quantum initializing state

$$U_{2n} = I - (1-i)\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i| \tag{4}$$

where I is a unity matrix and $|x_i\rangle$ is the index of the pattern and spurious states. This operator inverts the phases of $2n$ indices of states with 90°, but the phases of other indices corresponding to other orthogonal states are still invariable.

Step 3 Constructing the other unitary operator

$$U_\varphi = (1+i)\,|\varphi_0\rangle\,\langle\varphi_0| - i\,I \tag{5}$$

where $|\varphi_0\rangle$ is the index of the quantum state after quantum encoding. This operator inverts the amplitudes of all indices about their average value.

Step 4 After the Step 2 and Step 3 are conducted $T \cong \frac{\pi}{4}\sqrt{N/n}$ $(N = 2^i)$ times iteratively, the process is over.

Step 5 Computing the probability of pattern recognition. The proposed method always can search the targets with the probability of 100%. Moreover, the more the targets, the better this method.

4.3 Proving the Unitarity of Operators

Formulas (4) and (5) must satisfy unitarity, or else cannot execute quantum computation. Now we prove the unitarity of formula (4), and formula (5) follows in the same way.

$$U_{2n} = I - (1-i)\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i| = I - (1 - e^{\frac{\pi}{2}i})\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i|,$$

$$U_{2n}^{+} = I - (1 - e^{-\frac{\pi}{2}i})\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i|,$$

$$U_{2n}^{+}U_{2n} = I - (1 - e^{-\frac{\pi}{2}i})\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i| - (1 - e^{\frac{\pi}{2}i})\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i| + (1 - e^{-\frac{\pi}{2}i})(1 - e^{\frac{\pi}{2}i})\left(\sum_{i=1}^{i=2n} |x_i\rangle\langle x_i|\right)^2 = 1.$$

Therefore the operator of formula (4) is the unitary operator.

5 Comparison with other Searching Methods

Supposing there are six identified patterns, i.e. $|0000\rangle$, $|0001\rangle$, $|0011\rangle$, $|0101\rangle$, $|0110\rangle$, $|0111\rangle$ ($n = 6$). Each pattern is denoted with four qubits, so there are in all 16 pattern states. We compare the proposed method with other methods in recognizing these 6 patterns simultaneously.

5.1 Multi-Pattern Searching Method

The authors presented a multi-pattern high-probable quantum search algorithm [11]. This algorithm is very effective in searching relatively little patterns and total pattern set. But his probability in recognizing a number of patterns is not very high, its working process is as follow:

Step 1 Preparing the original state

$$|\varphi_0\rangle = \frac{1}{\sqrt{13}}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0).$$

We add 7 spurious states in the $|\varphi_0\rangle$ and the distribution of spurious states is also arbitrary.

Step 2 Inverting the phase of n indices of pattern states, but the phases of other indices corresponding to other orthogonal states are still invariable. i.e.:

$$|\varphi_1\rangle = \frac{1}{\sqrt{13}}(-1, -1, 1, -1, 1, -1, -1, -1, 1, 1, 1, 1, 1, 0, 0, 0).$$

Step 3 Inverting the amplitudes of all indices about their average value

$$|\varphi_2\rangle = \frac{1}{8\sqrt{13}}(9, 9, -7, 9, -7, 9, 9, 9, -7, -7, -7, -7, -7, 1, 1, 1).$$

Step 4 Inverting the phase of $2n$ indices of pattern states and spurious states

$$|\varphi_3\rangle = \frac{1}{8\sqrt{13}}(-9, -9, 7, -9, 7, -9, -9, -9, 7, 7, 7, 7, 7, 1, 1, 1).$$

Step 5 Inverting the amplitudes of all indices about their average value again

$$|\varphi_4\rangle = \frac{1}{32\sqrt{13}}(35, 35, -29, 35, -29, 35, 35, 35, -29, -29, -29, -29, -29, -5, -5, -5).$$

Step 6 After the Step 2 and Step 3 are conducted $T = \frac{\pi}{4}\sqrt{\frac{N}{n}} = \frac{\pi}{4}\sqrt{\frac{16}{6}} \approx 1$ times iteratively, the process is over (due to $T = 1$ the algorithm need not go on).

Step 7 Computing the probability of pattern recognition

$$p = 6 \times \left(\frac{35}{32\sqrt{13}}\right)^2 = 55.21\%.$$

5.2 Grover Algorithm Searching Multi-Pattern

Step 1 Initializing state by applying $H^{(4)} = H \otimes H \otimes H \otimes H$ to the state $|0000\rangle$ and getting an equiprobable superposition of all entries:

$$|\varphi_0\rangle = \sum_{x=0}^{2^4} |x\rangle = \frac{1}{4}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1).$$

Each index corresponds to its quantum state. For example, the first 1 represents the index of the state$|0000\rangle$.

Step 2 Inverting the phase of pattern states

$$|\varphi_1\rangle = \frac{1}{4}(-1, -1, 1, -1, 1, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1).$$

Step 3 Inverting the amplitudes of all indices about their average value

$$|\varphi_2\rangle = \frac{1}{8}(3, 3, -1, 3, -1, 3, 3, 3, -1, -1, -1, -1, -1, -1, -1, -1).$$

Step 4 After the Step 2 and Step 3 are conducted $T = \frac{\pi}{4}\sqrt{\frac{N}{n}} = \frac{\pi}{4}\sqrt{\frac{16}{6}} \approx 1$ times iteratively, the process is over (due to $T = 1$ the algorithm need not go on).

Step 5 Computing the probability

$$p = 6 \times \left(\frac{3}{8}\right)^2 = 84.375\%.$$

5.3 Searching Method with the Probability of 100%

This method that uses two operators of formula (4) and formula (5) and adopts special encoding strategy of original state can recognize multi-pattern with the probability of 100%. For this case, idiographic method is as follow:

Step 1 Producing initial quantum state and it is not linear superposition of all entries (seeing Sect. 4):

$$|\varphi_0\rangle = \frac{1}{\sqrt{12}}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0).$$

Step 2 Inverting the phase of state in the pattern set with 90° by using the unitary operator of formula (4)

$$|\varphi_1\rangle = \frac{1}{\sqrt{12}}(i, i, 1, i, 1, i, i, i, 1, 1, 1, 1, 0, 0, 0, 0).$$

Step 3 Inverting the amplitudes of all states about their average value and getting new state

$$(1+i)\,|\varphi_0\rangle\,\langle\varphi_0| - iI = \begin{bmatrix} (1-11i)/12 & (1+i)/12 & \cdots & 0 \\ (1+i)/12 & (1-11i)/12 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -i \end{bmatrix},$$

$$\frac{1}{\sqrt{12}}\begin{bmatrix} (1-11i)/12 & (1+i)/12 & \cdots & 0 \\ (1+i)/12 & (1-11i)/12 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -i \end{bmatrix}\begin{bmatrix} i \\ i \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ \cdots \\ b \\ 0 \end{bmatrix},$$

$$a = \frac{1}{\sqrt{12}}\left[\frac{1-11i}{12}\cdot i + \frac{1+i}{12}\cdot i\cdot 5 + \frac{1+i}{12}\cdot 6\right] = \frac{1}{\sqrt{12}}(1+i),$$

$$b = \frac{1}{\sqrt{12}}\left[\frac{1-11i}{12} + \frac{1+i}{12}\cdot i\cdot 6 + \frac{1+i}{12}\cdot 5\right] = 0.$$

Therefore new state is:

$$|\varphi_2\rangle = \frac{1}{\sqrt{12}}(1+i, 1+i, 0, 1+i, 0, 1+i, 1+i, 1+i, 0, 0, 0, 0, 0, 0, 0, 0).$$

Step 4 After the Step 2 and Step 3 are conducted $T = \frac{\pi}{4}\sqrt{\frac{N}{n}} = \frac{\pi}{4}\sqrt{\frac{16}{6}} \approx 1$ times iteratively, the process is over (due to $T = 1$ the algorithm need not go on).

Step 5 Computing the recognized probability

$$p_a = 6\cdot\left[\left(\frac{1}{\sqrt{12}}\right)^2 + \left(\frac{1}{\sqrt{12}}\right)^2\right] = 100\%, \qquad p_b = 0.$$

If we use the linear superposition of all entries in designing original state, i.e.:

$$|\varphi_0\rangle = \frac{1}{4}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1).$$

After the above same searching process, the searching probability has only:

$$p_a = 6 * \left[\left(\frac{5}{16}\right)^2 + \left(\frac{4}{16}\right)^2\right] = 96.09\%, \qquad p_b = 10 * \left(\frac{1}{16}\right)^2 = 3.91\%.$$

So there appears search result with the probability of 100% impossibly.

6 Conclusion and Prospect

This paper proposes a multi-pattern recognition method with the probability of 100% based on the improved Grover's algorithm. This method that adopts reasonable quantum coding to learn pattern set can recognize patterns with the probability of 1 after a series of unitary operations. After comparison with other quantum pattern recognition methods, their identified probability cannot reach 100%, which shows that the significance of the proposed method in improving the search accuracy is great. Unfortunately, this method is relatively complex in producing quantum original state. Moreover, the more the number of the patterns, the more the quantum bits, which makes that the number of quantum original states is very huge. How to solve this issue is our future work. Yet this method full makes advantage of the quantum parallel computing and can recognize multi-pattern simultaneously using quantum mechanics, which provides a new way in investigating pattern recognition.

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